1 HW 2

1.1 Exercise 1

Suppose that time to event X_i is distributed iid exponential with rate λ and time to right censoring C_i is distributed iid expontial with rate θ. Let $T_i = \min(X_i, C_i)$ and let $\delta_i = \mathbb{1}(X_i \le C_i)$. Let $X_i \perp\!\!\!\perp C_i$.

1. Find $P(\delta_i = 1)$.
2. Find the distribution of T_i.
3. Are T_i and δ_i independent? Show why or why not.
4. Let $(T_i, \delta_i), i = 1, \ldots, n$ be a sample from the above probability model. Find the MLE for λ, and find the mean and variance of the MLE.

1.2 Exercise 2

Suppose $X_i \overset{\text{iid}}{\sim} \text{Weibull}(\beta, \alpha)$ so that the hazard function is $\beta\alpha x^{\alpha-1}$, and suppose that C_i are independent of X_i and also iid. Let the censored observations be $T_i = \min(X_i, C_i)$ and $\delta_i = \mathbb{1}(X_i \le C_i)$ and suppose the set of realized data is given as $\{(t_i, \delta_i), i = 1, \ldots, n\}$.

1. Find the MLE for β when α is known
2. Find the MLE for α when β is known (you can give the answer as an implicit solution to an equation)

1.3 Exercise 3

This exercise involves finding the MLEs for α and β in a Weibull distribution from simulated data using the `survival` package in R. You will simulate a dataset $X_i, i = 1, \ldots, 1{,}000$ where X_i has a Weibull distribution with hazard function $\beta\alpha x^{\alpha-1}$, $\alpha = 2$ and $\beta = 1/2$

1. Read the R documentation to find out how base `R` parameterizes the Weibull distribution in its function `rweibull`, derive the map between our parameters and R's `shape` and `scale` parameters.
2. Simulate 1,000 observations from the Weibull with $\alpha = 2, \beta = \frac{1}{2}$.
3. Generate 1,000 C_i from an exponential distribution with the `rate` argument set to 1/3.

4. Create the vectors `delta` and `t`, with elements `delta`$[i] = \delta_i$ and `t`$[i] = t_i$ using our standard definitions shown above.

5. Use (t_i, δ_i) and the `survival` package in R (you may need to install it using the command `install.packages("survival")`), specifically, the function `survreg` to find the MLE of the Weibull parameters. This can be done with the following command:

```
survreg(Surv(t,delta,type = "right") ~ 1, dist="weibull")
```

6. Read the `survival` documentation to find the parameterization of the Weibull distribution and derive a map from `survival`'s Weibull distribution to `shape` and `scale`, and to α and β.

7. Plot the MLE for the survival function and overlay the true survival function using `pweibull`.

8. Using any method you like (delta method, bootstrap, etc.), plot the 95% pointwise confidence intervals for the survival function on the same plot.

Submit all your code for exercise 3 along with your rendered plots.

1.4 Exercise 4

Let X_i be distributed according a piecewise constant hazard function defined as follows:

$$\lambda_X(t) = \begin{cases} \lambda_1 & 0 \le t < t_1 \\ \lambda_2 & t_1 \le t < t_2 \\ \lambda_3 & t_2 \le t < \infty \end{cases}$$

1. Write the survival function for X_i

2. Suppose we have censored data as in Exercise 2. Find the MLE for $\lambda_1, \lambda_2, \lambda_3$.